Math 10A
Worksheet, Discussion #16; Monday, 7/9/2018
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1 Anti-Derivatives

1. True **FALSE** Just like differentiation where we can use the chain rule/product rule/quotient rule/etc. to always be able to find the derivative of a function, we can find similar rules to do the same with finding an anti-derivative.

2. True **FALSE** There exists a unique anti-derivative.

3. Find an antiderivative of $\frac{1}{2x}$.

> **Solution:** By the constant integration law, we know that an antiderivative of $\frac{1}{2x}$ is half an antiderivative of $\frac{1}{x}$, thus it suffices to find an antiderivative of $\frac{1}{x} = x^{-1}$. Ideally, we would take $\frac{x^{n+1}}{n+1}$ for an antiderivative of x^n, but $-1+1 = 0$ so we cannot use the power antiderivative rule. But, we remember that the derivative of $\ln x$ is $\frac{1}{x}$. So, an antiderivative of $\frac{1}{2x}$ is $\frac{\ln x}{2}$.

4. Find an antiderivative of $5e^x$.

> **Solution:** An antiderivative of e^x is e^x and so by the constant integration law, an antiderivative of $5e^x$ is $5e^x$.

5. Find an antiderivative to e.

> **Solution:** We can write $e = e \cdot x^0$. An antiderivative to x^0 is $x+1$ so by the constant antiderivative law, an antiderivative to e is $e(x+1) = ex + e$.

6. Find an antiderivative of $x + \sqrt{x}$.

Solution: An antiderivative of x is $\frac{x^2}{2}+5$ and an antiderivative of $\sqrt{x}=x^{1/2}$ is $\frac{2}{3}x^{3/2}+10$ and so by the addition integration law, an antiderivative of $x+\sqrt{x}$ is $\frac{x^2}{2}+\frac{2}{3}x^{3/2}+15$.

7. Find an antiderivative to $8t^3+15t^2$.

Solution: An antiderivative of t^3 is $\frac{t^4}{4}$ and an antiderivative of t^2 is $\frac{t^3}{3}+\pi$. So using the constant and addition antiderivative law, we get that an antiderivative of $8t^3+15t^2$ is $8\frac{t^4}{4}+15\frac{t^3}{3}+15\pi=2t^4+5t^3+15\pi$.

8. Find an antiderivative to $\cos u$.

Solution: One choice is $\sin u+5$.

9. Find an antiderivative to $\sin(2t)$.

Solution: We want to guess $-\cos(2t)$ but the derivative of $-\cos(2t)$ is $\sin(2t)\cdot 2$ by the chain rule. So we can multiply by one half to get a function that works. So one choice is $\frac{-\cos(2t)}{2}$.

10. Find the indefinite integral $\int(4t^3+3t^2)dt$.

Solution: The indefinite integral is

$$\int(4t^3+3t^2)dt=t^4+t^3+C.$$

11. Find the indefinite integral $\int\frac{1}{3x}dx$.

Solution: The indefinite integral is

$$\int\frac{1}{3x}dx=\frac{1}{3}\int\frac{1}{x}dx=\frac{\ln|x|}{3}+C.$$

2 Fundamental Theorem of Calculus I

2.1 Concept

12. If F is an antiderivative for f on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.

2.2 Problems

13. Evaluate the integral $\displaystyle\int_2^5 (x^2 + 1)dx$.

Solution: An antiderivative of $x^2 + 1$ is $\frac{x^3}{3} + x = F(x)$. So

$$\int_2^5 (x^2 + 1)dx = F(5) - F(2) = \frac{125}{3} + 5 - \frac{8}{3} - 2 = \frac{117}{3} + 3 = 39 + 3 = 42.$$

14. Evaluate the integral $\displaystyle\int_0^4 \sqrt{x}dx$.

Solution: We have that $\displaystyle\int_0^4 \sqrt{x}dx = \frac{2}{3} \cdot x^{3/2} \mid_0^4 = \frac{2}{3}(8 - 0) = \frac{16}{3}$.

15. Evaluate the integral $\displaystyle\int_1^8 \sqrt[3]{x}dx$.

Solution: We have

$$\int_1^8 \sqrt[3]{x} = \frac{3}{4}x^{4/3} \mid_1^8 = \frac{3}{4}(16 - 1) = \frac{45}{4}.$$

16. Evaluate the integral $\displaystyle\int_0^1 e^{x+1}dx$.

Solution: An antiderivative of e^{x+1} is itself so we can take

$$\int_0^1 e^{x+1}dx = e^{x+1} \mid_0^1 = e^2 - e^1 = e^2 - e.$$